THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

Screening Test – Ramanujan Contest

NMTC at INTER LEVEL – XI & XII Standards

2024 – 2025

Instructions:

1. Fill in the response sheet with your Name, Class and the and the institution through which you appear in the specified places.
2. Diagrams are only visual aids; they are NOT drawn to scale.
3. You are free to do rough work on separate sheets.
4. Use of Electronic gadgets such as Calculator, Mobile Phone or Computer is not permitted.
5. Duration of the test: 10 am to 12 noon (Two hours).
6. For each correct response you get 1 mark; for each incorrect response, you lose $1/2$ mark.

PART – A

Note

- Only one of the choices A, B, C, D is correct for each question. Shade the alphabet of your choice in the response sheet. If you have any doubt in the method of answering, seek the guidance of the supervisor.
- For each correct response you get 1 mark. **For each incorrect response you lose $\frac{1}{2}$ mark.**

1. Let $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}=\frac{m}{n}$, where m and n are positive integers with no common divisors other than 1. The highest power of 7 that divides m is

   A. 0 B. 1 C. 2 D. 3

2. Five spherical balls of diameter 10 cm each fit inside a closed cylindrical tin with internal diameter 16 cms. What is the smallest height of the tin can be?

   A. 39 B. 42 C. 45 D. 48

3. The expression $\dfrac{7n+18}{2n+3}$ takes integer values for certain integer values of n. The sum of all such values of the expression is

   A. 14 B. 21 C. 24 D. 28

4. P is a point inside $ABCD$ such that $PA=2, PB=4, PC=5$ and $PD=6$. The maximum area of the quadrilateral $ABCD$ is

   A. 30 B. 33 C. 35 D. 38

5. The value of

$$(3^{4/3}-3^{1/3})^3+(3^{5/3}-3^{2/3})^3+(3^{6/3}-3^{3/3})^3+\cdots+(3^{10/3}-3^{7/3})^3$$

   is

   A. $12(3^7-1)$ B. $12(3^7+1)$ C. $6(3^7-1)$ D. $6(3^7+1)$

6. One hundred people are standing in a line and they are required to count off in fives as "one, two, three, four, five" and so on from the first person in the line. Anyone who counts "five" walks out of the line. Those remaining repeat this procedure until only four people remain in the line. What was the original position in the line of the last person to leave?

   A. 94 B. 96 C. 97 D. 98

7. The number of values of positive integers n for which $1! + 2! + \cdots + n!$ is a perfect square is

A. 0 B. 1 C. 2 D. Infinitely many

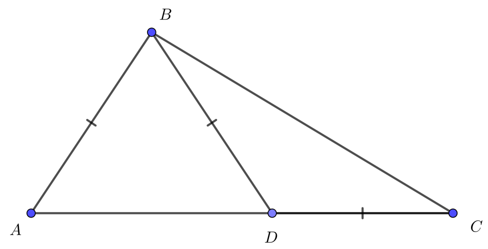
8. When $2025^{2026} - 2025$ is divided by $2025^2 + 2026$, the remainder is

A. 0 B. 2025 C. 2026 D. None of these

9. For a real number x, let $\lfloor x \rfloor$ denote the largest integer $\leq x$. For example, $\lfloor 3.4 \rfloor = 2$ and $\lfloor 4.9 \rfloor = 4$. Let $N = \lfloor (\sqrt{27} + \sqrt{23})^6 \rfloor$. The remainder when N is divided by 1000 is

A. 799 B. 599 C. 399 D. 199

10. ABC is a triangle. D lies on AC such that $AB = BD = CD$. All the angles in the diagram are a positive whole number of degrees. The largest possible size, in degrees of $\angle ABC$ is



A. 171 B. 173 C. 175 D. 177

11. The side lengths of a right angled triangle are in geometric progression and the smallest side has length 2 units. The length of the hypotenuse is

A. $1 + \sqrt{5}$ B. $\sqrt{10}$ C. $3\sqrt{2} - 1$ D. $\sqrt{11}$

12. In a regular polygon there are two diagonals that intersect inside the polygon at an angle $50°$. The least number of sides of the polygon for which this is possible is

A. 12 B. 18 C. 24 D. 36

13. In triangle PQR, $\angle R = 2\angle P$, $PR = 5$ and $QR = 4$. The length of PQ is

A. $2\sqrt{10}$ B. 6 C. 7 D. $2\sqrt{7}$

14. Each of ten people around a circle chooses a number and tells it to the neighbor on each side. Thus each person gives out one number and receives two numbers. The players then announce the average of the two numbers they received. The announced numbers, in order around the circle were $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$. The number chosen by the person who announced the number 6 is

A. 7 B. 5 C. 3 D. 1

15. A regular octagon is formed by cutting off four equal isosceles right angled triangles from the corners of a square of side length 1. The area of the octagon is

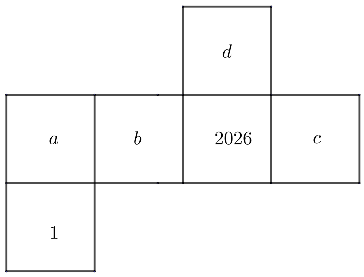
A. $2(\sqrt{2} - 1)$ B. $4\sqrt{2} - 3$ C. $\sqrt{2} - 1$ D. $3\sqrt{2} - 2$

PART – B

Note

- Write the correct answer in the space provided in the response sheet.
- For each correct response you get 1 mark. **For each incorrect response you lose $\frac{1}{4}$ mark.**

16. The diagram shows the net of a cube, that is, we can fold along the edges of the squares to make a cube from this net. On each face there is an integer written $-\ 1, a, b, c, d, 2026$. If each of the



four numbers a, b, c, d equals the average of the numbers on the four faces of the cube adjacent to it, the value of a is ———.

17. Let $S = \{1, 2, 3, \ldots, 15\}$. The number of subsets A of S containing four elements such that any two elements of A differ by at least 2 is ———.

18. Given a deck of 52 cards with numbers $1, 2, \ldots, 52$ written on them, one number per card. The deck is shuffled and 13 cards are chosen at random form the shuffled deck and thrown away, without noting the numbers on them. From the remaining 39 cards, one card is chosen at random. If the probability that the number on this card is a multiple of 13 is $\frac{m}{n}$, where m, n are integers with no common divisor other than 1, then $m + n$ equals ———.

19. Suppose 10 objects are placed along a circle at equal distances. The number of ways can three objects be chosen from among them so that no two of the chosen objects are adjacent or diametrically opposite is ———.

20. For a positive integer n, let $n \mod 13$ denote the remainder r, $0 \le r < 13$ when divided by 13. If a, b, c are integers such that

$$\begin{aligned} 4a + 5b + 6c &= 1 \mod 13 \\ a - b - 7c &= 3 \mod 13 \\ 3a - 4b + 5c &= 9 \mod 13 \end{aligned}$$

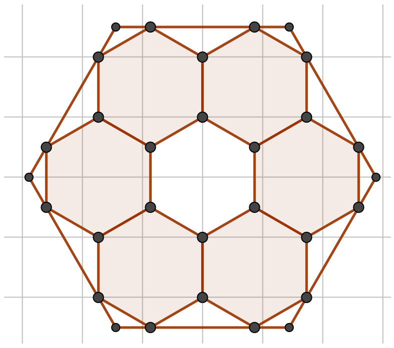
then $a + b + c \mod 13$ is ———.

21. The sum of all positive integers N less than 2024 such that N equals 13 times the sum of digits of N (when N is written in base 10) is ———.

22. Six identical regular hexagons are arranged inside a larger hexagon as shown in the Figure. The outer hexagon has area 900 square units. The area of the shaded region (the total area contained in the smaller hexagons) is ——— square units.

23. A positive integer is said to be *special* if the sum of the remainders obtained when it is divided by five consecutive positive integers is 32. For example, 24 is special since when divided by $11, 12, 13, 14, 15$ the remainders are $2, 0, 11, 10, 9$ and these add up to 32. The smallest positive integer that is special is ———.

24. The largest three digit number with the property that the number is equal to the sum of the hundreds digit, the square of its tens digit and the cube of its units digit is ———.



25. It is a surprising fact that $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 8 \times 9 \times 10$. It is more surprising that

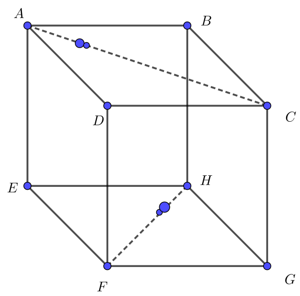
$$8 \times 9 \times 10 \times 11 \times 12 \times 13 \times 14$$

can be written as a product of consecutive positive integers. The smallest number in the product is ———.

26. There are 100 points $P_1, P_2, \ldots, P_{100}$ placed on a line such that the distance between P_i and P_{i+1} is $\frac{1}{i}$ for $1 \leq i \leq 99$. The sum of the distances between every pair of these points is ———.

27. For any positive integer n, let $d(n)$ denote the number of divisors of n. For example, $d(4) = 3$, since the divisors of 4 are $1, 2, 4$. The smallest positive integer n for which $d(n-2) + d(n) + d(n+2) = 21$ is ———.

28. Two bugs sit at the vertices A and H of a cube $ABCDEFGH$ with edge length $4\sqrt{110}$ units. The bugs start moving simultaneously along AC and HF with the speed of the first bug twice that of the other one. The shortest distance between the bugs is ———.



29. The smallest positive integer n for which it is possible to draw an n-gon whose vertex angles all measure $163°$ or $171°$ is ———.

30. Let $P(x) = ax^3 + bx^2 + cx + d$ be a cubic polynomial such that $P(2) = 7, P(3) = 13$ and $P(5) = 7$. If the sum of the three roots of $P(x) = 0$ is 40, the value of $P(35)$ is ———.